Principle

A sinusoidal voltage is applied to a non-linear circuit with a silicon diode functioning as voltage-independent capacity. The oscillating circuit, i.e. the occurrence of chaotic oscillatory behaviour with increasing amplitude is studied.

Related topics

Oscillating circuit, forced oscillation, diode, period multiples, Fourier spectrum, chaotic oscillation

Tasks

Study the oscillatory behaviour of the non-linear circuit for an exciting signal with amplitudes between 0.1 and 15 V at frequency of 35 kHz. Investigate the oscillatory image and the Fourier spectrum as functions of the excitation amplitude. Show that period multiples and regions of chaotic oscillatory amplitude occur with increasing amplitude.

Fig 1: Experimental set-up.
Forced oscillations of a non linear electrical series resonant circuit – chaotic oscillation

Equipment

1  Cobra3 Basic Unit, USB  12150-00
1  Cobra3 power supply  12151-99
1  Data cable USB  14608-00
1  Software Cobra3, Frequency Analysis  14514-61
1  Digital function generator  13654-99
1  Oscilloscope, 30 MHz, 2 channels  11459-95
1  Connection box  06030-23
1  Coil, 1200 turns  06515-01
1  Clamping holder  06049-15
1  Connecting plug  39170-00
1  Connector, T type, BNC  07542-21
1  Adapter, BNC socket, 4 mm socket pair  07542-26
1  Adapter, BNC socket, 4 mm plug pair  07542-27
1  Connecting cord, blue, l = 100 mm  07359-04
2  Connecting cord, red, l = 250 mm  07360-01
2  Connecting cord, red, l = 500 mm  07361-01
2  Connecting cord, blue, l = 500 mm  07361-04
1  Screened cable, BNC, l = 250 mm  07542-10
1  Screened cable, BNC, l = 750 mm  07542-11
1  PC, Windows® 95 or higher

Set-up

Set up the series circuit according to Figs. 1 and 2. The input voltage is tapped off at the diode and fed to the ANALOG IN 2 (+,−) input jacks of the Cobra3 Basic Unit and the oscilloscope channel CH2. Connect the circuit with the power output of the digital function generator and observe the chaotic oscillations.
generator. Take care to connect the ground to the diode. Use the T connection to connect the power output with the oscilloscope input jack CH2, too.

**Procedure**

Use a sinusoidal voltage signal as input with amplitude of 0.1 V. For all measurements use a frequency of 35 kHz.

Activate the *Frequency Analysis* program module. Choose the continuous measuring mode and the maximum frequency range. For each measurement set the voltage range to an appropriate value.

In the course of this experiment increase the input voltage of the digital function generator progressively from 0.1 to the maximum value in appropriate steps. Observe both signals on the oscilloscope and the PC screen and record both the signal and the Fourier spectrum for each step. For regions of chaos record several spectra for the same amplitude in order to verify that the occurring fundamental frequencies vary in number and value.

**Theory**

As a first approximation, the characteristic of a diode behaves quadratic with amplitude of the input signal. The diode's barrier layer, which is responsible for its capacitance, changes as a function of the applied voltage and the voltage's sign. Due to the quadratic characteristic line, the behaviour becomes nonlinear. This is responsible for the onset of chaos. This behaviour can be described by a bifurcation diagram, which is based on a quadratic function. In Fig. 4 such a diagram is shown which describes the “path to chaos” for this experiment. The x-axis shows the increasing voltage of the input signal, the y-axis denotes the frequencies. For an increased voltage an increasing quantity of frequencies...
may be observed. After exceeding a certain threshold the behaviour of the oscillating circuit can no longer be predicted – it behaves chaotically.

![Logarithmic bifurcation diagram](image)

Fig 4: Logarithmic bifurcation diagram

**Note:**

The fundamental theoretical framework for chaos theory can naturally not be presented here. However, it has been exhaustively explained in many publications:


**Evaluation and results**

In the following the evaluation of the obtained values is described with the help of example values. Your results may vary from those presented here.

At very low voltages the diode does not show its rectifying effect and functions as a pure capacitance. Accordingly, the circuit oscillates like the generator, namely sinusoidal as shown in Fig. 5. Only a single line appears in the Fourier spectrum at the frequency of 35.2 kHz which corresponds to the natural frequency of the oscillating circuit. With further increased amplitude of the exciting signal the diode only allows a single half-wave of the alternating signal to pass (Fig. 6). According to the theory of Fourier analysis, this means a spectrum with discrete harmonic lines whose lowest frequency is the fundamental
oscillation. These conditions of the oscillatory circuit are far to the left in the bifurcation diagram in Fig. 4.

**Fig 5:** Voltage tapped off the diode at $U_{ss} = 0.1 \text{ V}$. The circuit oscillates sinusoidally as the diode is functioning as pure capacitance.

**Fig 6:** Voltage tapped off the diode at $U_{ss} = 1.0 \text{ V}$. The diode now shows its rectifying effect.

In Fig. 7 the oscillating circuit exhibits the behaviour occurring in the bifurcation diagram at the first fork. On the oscilloscope screen one signal of higher amplitude and one of lower amplitude are alternately displayed (depending on the trigger settings). This corresponds to a period doubling: the 17.6 kHz line appears in the spectrum as the fundamental frequency.
In Fig. 8 an additional period doubling occurs. The fundamental frequency of the spectrum is now at 8.8 kHz. On the oscilloscope screen a continually increasing ramification of the periodicity is observed with every additional increase of signal amplitude. In the bifurcation diagram the second splitting has been reached.

Fig. 7: Voltage tapped off the diode at $U_{ss} = 3.0 \, \text{V}$. The first period doubling appears. The fundamental frequency is halved.

Fig. 8: Voltage tapped off the diode at $U_{ss} = 7.0 \, \text{V}$. The second period doubling (period quadrupling) appears. The fundamental frequency is a quarter of the natural frequency.
In Fig. 9 the chaotic status has been achieved. Due to the rectifying effect of the diode, harmonic lines still appear in the Fourier spectrum, but in contrast to the cases above frequencies between these lines appear continuously with changing partial amplitudes. The shape of the Fourier spectrum changes continuously and no two identical measurements for the same signal amplitude can be taken. The circuit oscillates with all the displayed frequencies – thus chaotically. The chaotic character of the oscillations also becomes clear when regarding the amplitudes of the recorded signal. Quite obviously, there is no periodicity at all in the sequence of the signal heights.

In Fig. 10 the chaotic status has been achieved. Due to the rectifying effect of the diode, harmonic lines still appear in the Fourier spectrum, but in contrast to the cases above frequencies between these lines appear continuously with changing partial amplitudes. The shape of the Fourier spectrum changes continuously and no two identical measurements for the same signal amplitude can be taken. The circuit oscillates with all the displayed frequencies – thus chaotically. The chaotic character of the oscillations also becomes clear when regarding the amplitudes of the recorded signal. Quite obviously, there is no periodicity at all in the sequence of the signal heights.

In Fig. 10 the oscillator has left the chaotic state for $U_{ss} = 16.0 \text{ V}$. These islands of order are also predicted by the bifurcation diagram. Only the first fundamental and its higher
harmonics show in the spectrum. The amplitudes of the signal are periodic. With further increase of the input signal the oscillator will slip back into chaos.

**Remarks on experimental procedure**

If the circuit in Fig. 3 is expanded according to the circuit diagram in Fig. 11, it is possible to hear the signals of the oscillators over a loudspeaker. To achieve this, the signal is tapped between the coil and the diode over a 1 kΩ resistor (39104-41) and led to a LF amplifier (13625-93). From the amplifier the signal passes to the loudspeaker (13765-00). A frequency between 8 and 9 kHz should be chosen for this experiment. At this frequency, the experiment does not follow the bifurcation diagram as at a frequency of 35 kHz. However, with the loudspeaker it is possible to make the period doubling (or frequency bisection) phenomenon audible.

The oscillator described here was experimentally investigated for the first time by Lindsay in 1981 (theory section and the given literature therein).

![Circuit diagram for acoustic reproduction of the occurring oscillations.](image)